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Hydromagnetic stability of dissipative flow between rotating permeable cylinders. Part 2. Oscillatory critical modes and asymptotic results

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In considering the onset of instability of an electrically conducting fluid between rotating permeable perfectly conducting cylinders in an applied axial magnetic field, it is found that oscillatory axially symmetric modes occur at large values of Hartmann number, in addition to the usual stationary modes. Results are presented showing the effect of the oscillatory modes on the criterion of onset of instability.

The asymptotic behaviour of the stability criterion is considered in the limit of very large radial Reynolds number, and also in the limit where both the radial Reynolds number and the Hartmann number are large.

1. Introduction

In part 1 of this paper (Chang & Sartory 1967*a*), the stability of the flow of an electrically conducting fluid between perfectly conducting rotating permeable cylinders under the influence of an axial magnetic field was considered for stationary or nonoscillatory axisymmetric critical modes. It is now known, however, from earlier work by the authors (1967*b*) on the stability of hydromagnetic Couette flow between *non*-permeable perfectly conducting cylinders, that oscillatory axisymmetric critical modes are also possible for sufficiently high values of the applied magnetic field or Hartmann number. It was therefore expected that oscillatory modes would also occur under certain conditions for flow between permeable cylinders. In the present work, the occurrence of oscillatory modes and their effect on the onset of instability of flow between permeable cylinders is considered.

Because of the large number of parameters influencing the stability of the type of flow considered in this work, it is difficult to cover a wide range of values of all of the parameters. One useful method of extending the range of the results is to derive simplified asymptotic or limiting forms of the stability equations. The solutions of the asymptotic forms of the equations are considered for both stationary and oscillatory critical modes.

2. Finite $|R_r|$ theory

2.1. *Formulation of the oscillatory mode problem*

As discussed in part 1, we make the approximations that the magnetic Prandtl number, P_m, and its product with the magnitude of the radial Reynolds number, $P_m|R_r|$, are both small. The stationary solution of the basic hydromagnetic equations under consideration is

$$\left.\begin{aligned} (u,v,w) &= [N'/r, L'/r + 2M'r^{R_r+1}/(R_r+2), 0], \\ (B_r, B_\theta, B_z) &= (0,0,B_0), \\ P_m\,(\text{magnetic Prandtl number}) &= \mu_0\sigma\nu, \\ R_r\,(\text{radial Reynolds number}) &= N'/\nu, \end{aligned}\right\} \tag{2.1}$$

where

(u, v, w), (B_r, B_θ, B_z) are the cylindrical components of the velocity and the magnetic induction field, respectively; r is the radial co-ordinate, ν is the kinematic viscosity, μ_0 is the magnetic permeability, σ is the electrical conductivity, L', M', N' are constants, B_0 is the applied magnetic field and the rationalized MKS system of units is used. The fluid is assumed to be bounded by concentric permeable perfectly conducting cylinders.

Under the above approximations, the linearized stability equations for the flow (2.1) with respect to small amplitude oscillatory modes with time, t, and axial, z, dependence of the form $\exp(ipt+ikz)$ reduce to

$$\left.\begin{aligned} &[\{DD_+ - \beta_2^2 - (R_r/x)D_- - ia_2\}\{DD_+ - \beta_2^2\} + \beta_2^2 Q_2]W_1 \\ &\qquad + \beta_2^2 T\{(1/x^2) + \lambda x^{R_r}\}\{DD_+ - \beta_2^2\}W_4 = 0, \\ &[\{DD_+ - \beta_2^2 - (R_r/x)D_+ - ia_2\}\{DD_+ - \beta_2^2\} + \beta_2^2 Q_2]W_4 - x^{R_r}W_1 = 0, \end{aligned}\right\} \tag{2.2}$$

where

$$W_1 = \frac{2M'r_2^{R_r+2}}{\nu}\frac{u'}{V_0},$$

$$W_4 = \frac{1}{kr_2}\frac{1}{V_0 r_2 \mu_0 \sigma}\frac{B'_\theta}{B_0},$$

$$a_2 = \frac{pr_2^2}{\nu}, \quad \beta_2 = kr_2,$$

$$x = r/r_2, \quad D = d/dx, \quad D_\pm = D \pm 1/x,$$

$$T\ (\text{Taylor number}) = -\frac{4L'M'r_2^{R_r+2}}{\nu^2},$$

$$Q_2\ (\text{Hartmann number})\dagger = \sigma B_0^2 r_2^2/(\nu\rho),$$

$$\lambda = \frac{2M'r_2^{R_r+2}}{L'(R_r+2)},$$

V_0 is a characteristic speed, ρ is the fluid density, r_2 is the radius of the outer cylinder, and u', B'_θ are the amplitudes of the radial r-component of the velocity perturbation and the transverse θ-component of the magnetic induction vector perturbation, respectively.

† In this paper we refer to the various quantities denoted by Q as Hartmann numbers, although Q actually has the form of the square of the usual Hartmann number.

The boundary conditions for perfectly conducting walls are

$$W_1 = DW_1 = D_+W_4 = (DD_+ - \beta_2^2)W_4 = 0 \quad \text{at} \quad x = \kappa, \quad 1{\cdot}0, \tag{2.3}$$

where $\kappa = r_1/r_2$, and r_1 is the inner cylinder radius.

The neutral Taylor number is a value dependent on β_2 for which (2.2) and (2.3) have a solution with $\mathrm{Im}\,(a_2) = 0$. The critical Taylor number is the minimum value of the neutral Taylor number for all values of β_2.

It was shown in part 1† that the curves of critical Taylor number and wave-number for all values of radius ratio and Hartmann number approach a common asymptote as $|R_r| \to \infty$ provided appropriate definitions of the parameters are adopted for inward and outward radial flow. For inward flow, we put

$$T_1 = (-\omega\Omega)_m r_1^4/\nu^2, \quad \beta_1 = kr_1, \quad Q_1 = \sigma B_0^2 r_1^2/(\nu\rho), \quad a_1 = pr_1^2/\nu; \tag{2.4}$$

and for outward flow,

$$T_2 = (-\omega\Omega)_m r_2^4/\nu^2, \quad \beta_2 = kr_2, \quad Q_2 = \sigma B_0^2 r_2^2/(\nu\rho), \quad a_2 = pr_2^2/\nu, \tag{2.5}$$

where
$$(-\omega\Omega)_m = \max_{r_1 \leqslant r \leqslant r_2} (-\omega\Omega), \tag{2.6}$$

with
$$\Omega(r) = v/r, \quad \omega(r) = 2(dv/dr + v/r). \tag{2.7}$$

2.2. *Finite $|R_r|$ results*

Equations (2.2) and (2.3) have been solved numerically using methods described in Chang & Sartory (1967*b*) and in part 1.

In part 1, it was found that at large values of the Hartmann number discontinuities in the critical wave-number occurred at certain values of the parameters. The discontinuities were found to be caused by shifts in the minimum between loops of the neutral Taylor number versus wave-number diagram. In the solid curves of figure 1, we reproduce a stationary mode neutral diagram showing several loops with competing minima. If the value of Q_2 were reduced continuously to zero, the right-most loop of figure 1 would map into a pair of nested U-shaped curves representing the two lowest normal modes. Under the conditions of figure 1, there is no Taylor number at which either of the two lowest modes can be made stationary for wave-numbers less than about 60. The dashed curve in figure 1 shows the effect of admitting oscillatory as well as stationary neutral modes. It represents a complex conjugate pair of oscillatory modes, and extends the neutral curve for the two lowest modes across the entire range of the wave-numbers. Similar extensions exist also for the partial loops representing higher pairs of modes, although the corresponding curves have not been calculated. The oscillatory portion of the neutral curve for the two lowest modes extends about an order of magnitude below the stationary mode curves, and determines the minimum or critical value of the Taylor number for the onset of instability.

Having established that oscillatory critical modes must be admitted to obtain the correct critical values, we now consider the effect of such modes on the results

† There were a few misprints in part 1. The symbols of the abscissas and ordinates should be interchanged and $B_{1,2}$ should be $\beta_{1,2}$ in figures 1–4 and 6, v should be divided by (R_r+2) in (2.1), and H^2 should be Q_2 in (2.12).

presented in part 1. Oscillatory modes do not occur for Hartmann numbers of 100 or less, so that we can restrict our attention to the higher values considered in part 1.

Figures 2–4 present the critical values of the Taylor number, wave-number, and frequency $\mathrm{Re}\,(a_2)$ as a function of the radial Reynolds number for $Q_2 = 10^3$, a non-rotating outer cylinder, and outward radial flow. Results are shown for $\kappa = 0{\cdot}25$ and $0{\cdot}4$. The third curve on each graph presents results of the asymptotic theory which will be discussed later. The discontinuities in figures 3 and 4

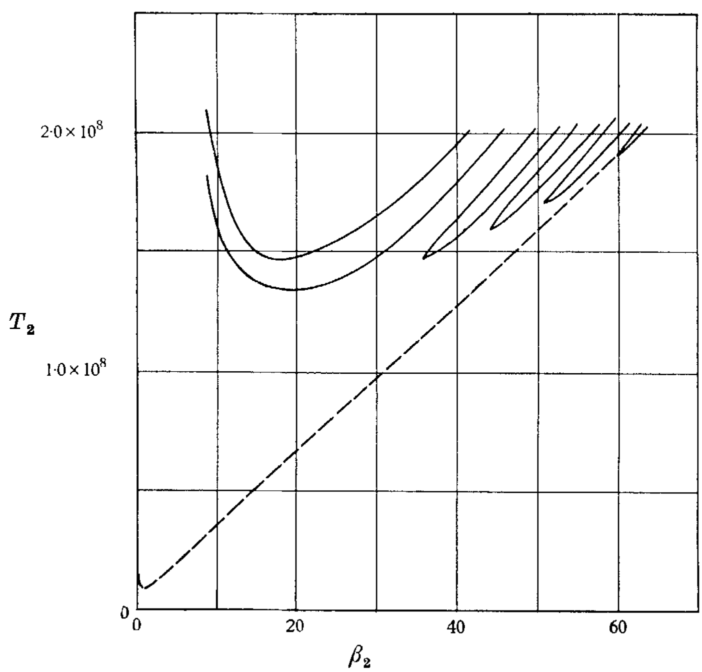

FIGURE 1. Neutral Taylor number versus wave-number for $\lambda = -1{\cdot}0$, $Q_2 = 10^4$, $\kappa = 0{\cdot}25$, $R_r = 2{\cdot}5$. ——, stationary modes; – – –, oscillatory modes.

represent transitions between oscillatory and stationary critical modes. In the interval between the discontinuities the critical modes are oscillatory as indicated by the non-zero frequency of figure 4. Outside of the interval the modes are stationary and the results reduce to those given in part 1.

The first transition from stationary to oscillatory modes is believed to be caused by changes in the shape of the primary transverse velocity profile produced by outward radial flow which favour the oscillatory modes. At high values of the radial Reynolds number the disturbance becomes confined to a boundary layer on the outer cylinder, and the Hartmann number based on the boundary-layer thickness eventually becomes too small to maintain oscillatory modes, so that a transition back to stationary modes results.

Figures 5–7 present results for the same conditions as above except that $Q_2 = 10^4$. The oscillatory modes now extend downward to $R_r = 0$, and the final transition to stationary modes is displaced to the right.

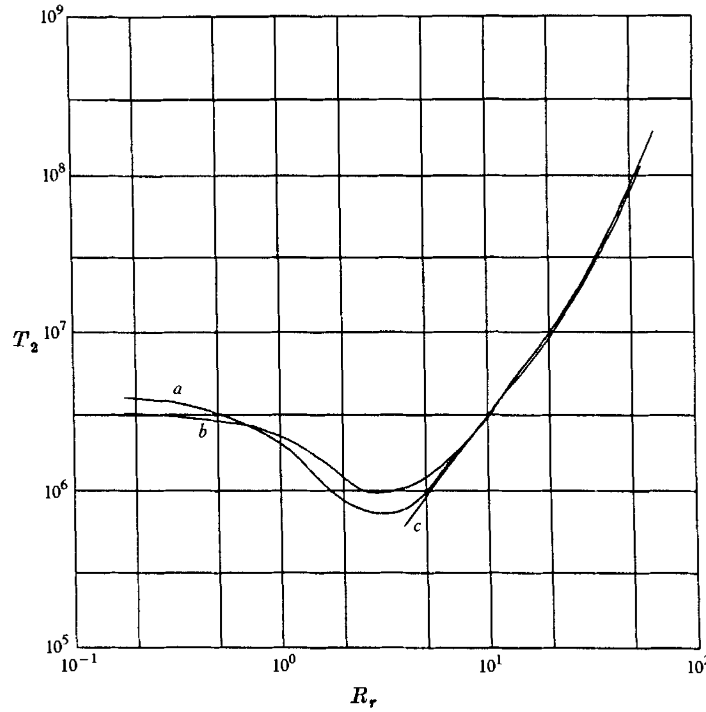

FIGURE 2. Critical Taylor number versus radial Reynolds number for $\lambda = -1{\cdot}0$, $Q_2 = 10^3$. (a) $\kappa = 0{\cdot}25$; (b) $\kappa = 0{\cdot}4$; (c) $|R_r| \to \infty$ asymptotic results.

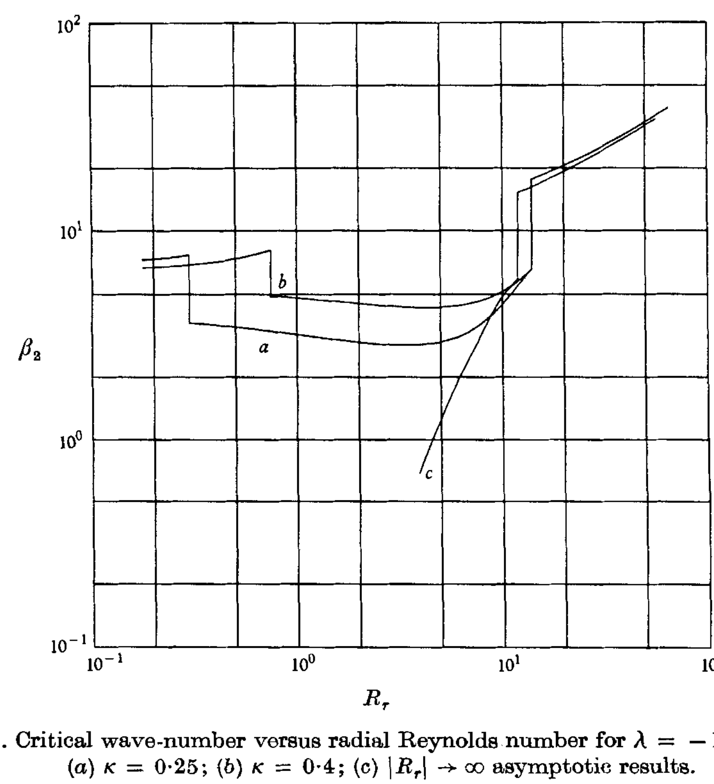

FIGURE 3. Critical wave-number versus radial Reynolds number for $\lambda = -1{\cdot}0$, $Q_2 = 10^3$. (a) $\kappa = 0{\cdot}25$; (b) $\kappa = 0{\cdot}4$; (c) $|R_r| \to \infty$ asymptotic results.

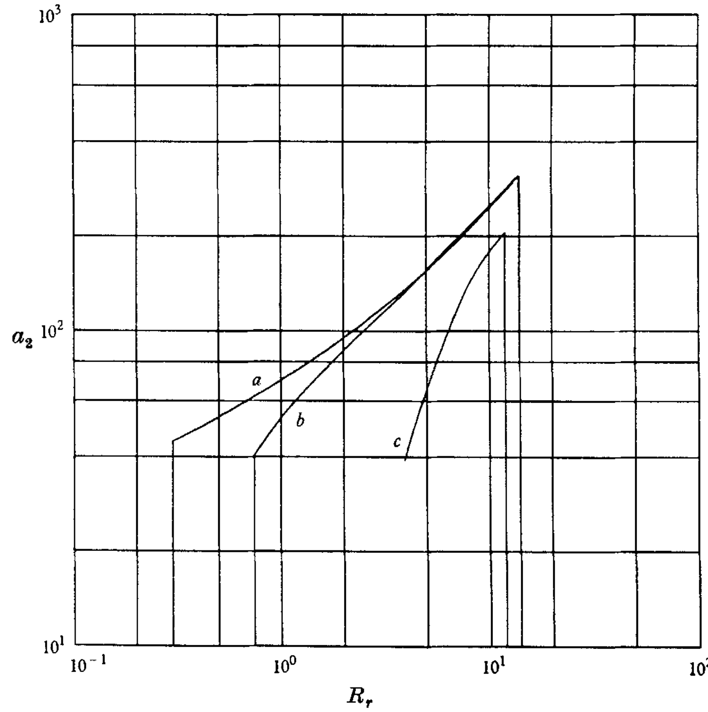

FIGURE 4. Critical frequency versus radial Reynolds number for $\lambda = -1{\cdot}0$, $Q_2 = 10^3$. (a) $\kappa = 0{\cdot}25$; (b) $\kappa = 0{\cdot}4$; (c) $|R_r| \to \infty$ asymptotic results.

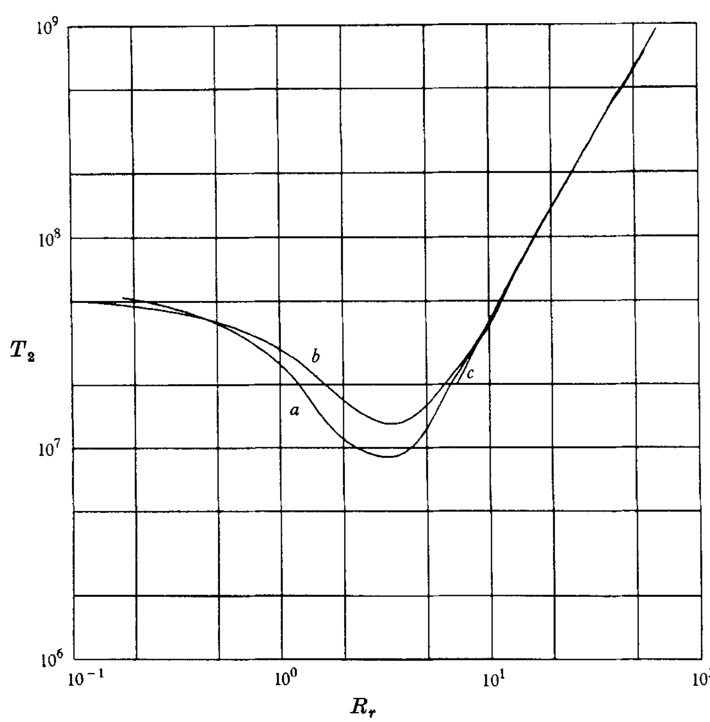

FIGURE 5. Critical Taylor number versus radial Reynolds number for $\lambda = -1{\cdot}0$, $Q_2 = 10^4$. (a) $\kappa = 0{\cdot}25$; (b) $\kappa = 0{\cdot}4$; (c) $|R_r| \to \infty$ asymptotic results.

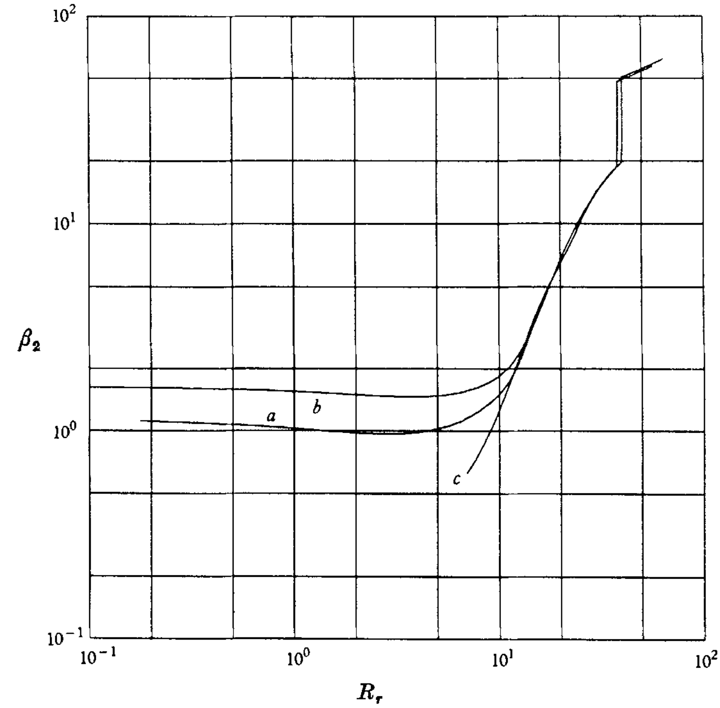

FIGURE 6. Critical wave-number versus radial Reynolds number for $\lambda = -1{\cdot}0$, $Q_2 = 10^4$. (*a*) $\kappa = 0{\cdot}25$; (*b*) $\kappa = 0{\cdot}4$; (*c*) $|R_r| \to \infty$ asymptotic results.

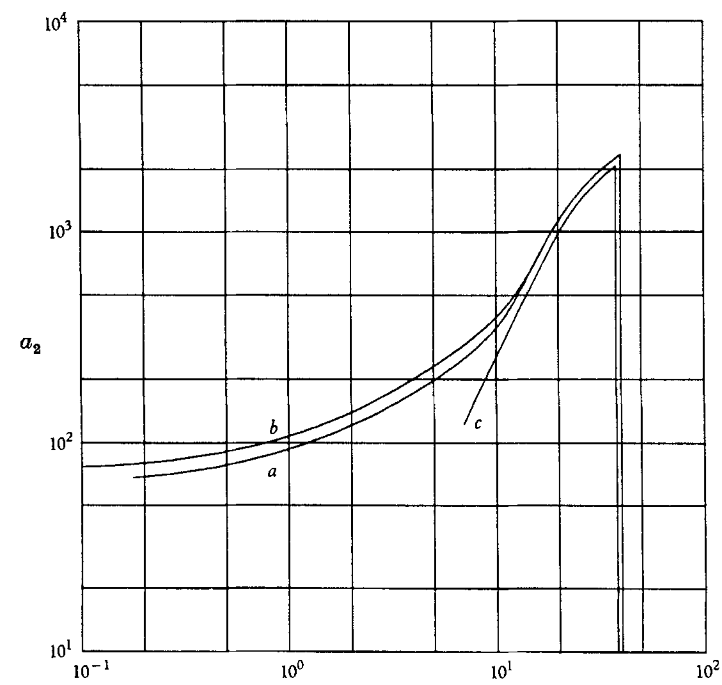

FIGURE 7. Critical frequency versus radial Reynolds number for $\lambda = -1{\cdot}0$, $Q_2 = 10^4$. (*a*) $\kappa = 0{\cdot}25$; (*b*) $\kappa = 0{\cdot}4$; (*c*) $|R_r| \to \infty$ asymptotic results.

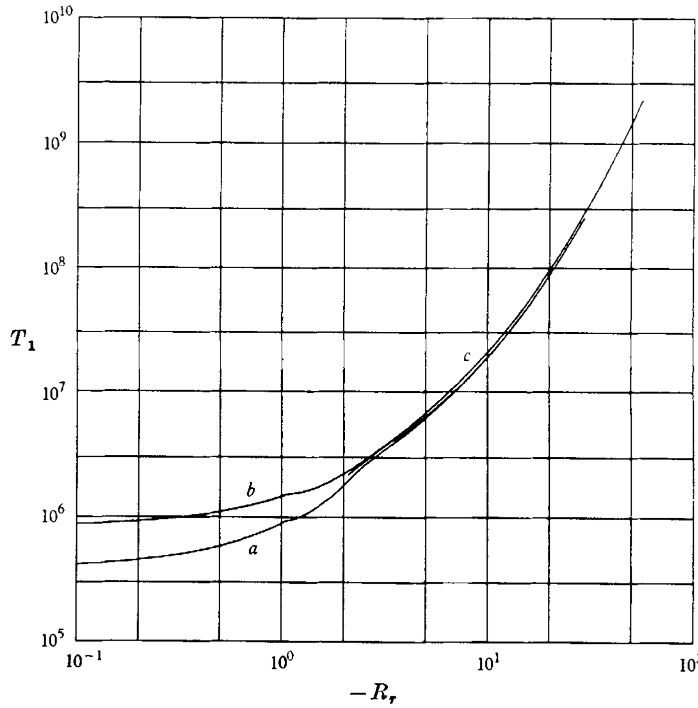

FIGURE 8. Critical Taylor number versus radial Reynolds number for $\lambda = -1{\cdot}0$, $Q_1 = 10^3$. (*a*) $\kappa = 0{\cdot}25$; (*b*) $\kappa = 0{\cdot}4$; (*c*) $|R_r| \to \infty$ asymptotic results.

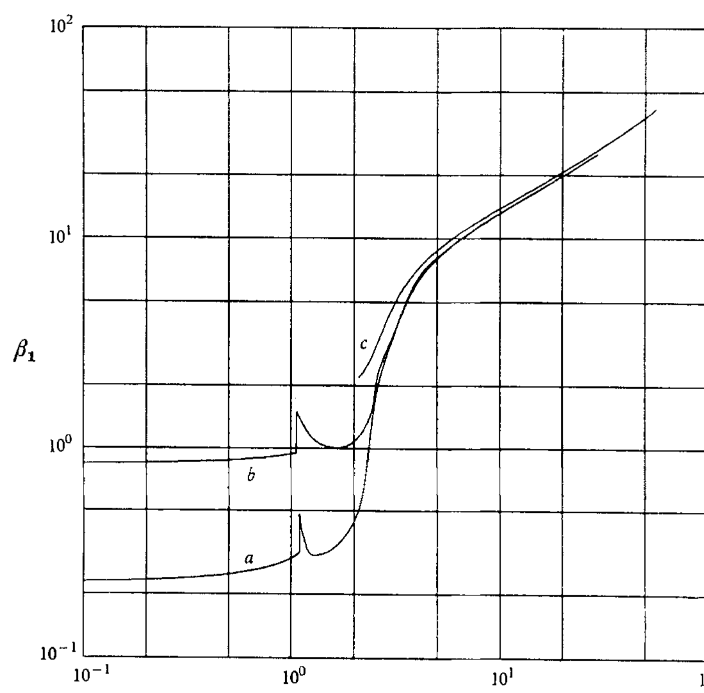

FIGURE 9. Critical wave-number versus radial Reynolds number for $\lambda = -1{\cdot}0$, $Q_1 = 10^3$. (*a*) $\kappa = 0{\cdot}25$; (*b*) $\kappa = 0{\cdot}4$; (*c*) $|R_r| \to \infty$ asymptotic results.

Figures 8–10 present results for $Q_1 = 10^3$ and a non-rotating outer cylinder; but for inward radial flow, $R_r < 0$. Here oscillatory modes are present down to $R_r = 0$. The very early transition back to stationary modes, $R_r \approx -1{\cdot}05$, is believed to occur because the changes in the shape of the primary transverse velocity profile produced by inward radial flow favour stationary modes. No appreciable boundary-layer effects exist at such a low radial Reynolds number.

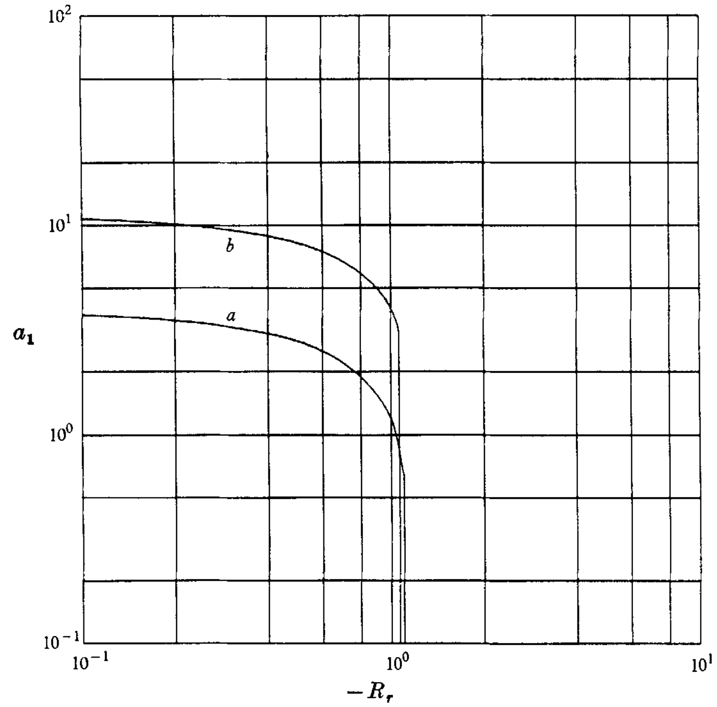

FIGURE 10. Critical frequency versus radial Reynolds number for $\lambda = -1{\cdot}0$, $Q_1 = 10^3$. (a) $\kappa = 0{\cdot}25$; (b) $\kappa = 0{\cdot}4$.

In part 1, results for a radius ratio of 0·8 were considered in addition to the values 0·4 and 0·25 used here. Results for $\kappa = 0{\cdot}8$ have not been included here because oscillatory modes do not occur for the range of the other parameters chosen. It should not be inferred that narrow gap widths inhibit the formation of oscillatory modes. On the contrary, it is known for the case $R_r = 0$ that oscillatory modes begin to form for $\kappa = 0{\cdot}8$ at somewhat lower values of Hartmann number than for $\kappa = 0{\cdot}4$ or $0{\cdot}25$ provided the Hartmann number is based on gap width rather than on cylinder radius. Thus, the absence of oscillatory modes for $\kappa = 0{\cdot}8$ is merely a result of the definition and range of values chosen for the Hartmann number in this work.

3. Asymptotic theory

3.1. *Stability equations for $|R_r| \to \infty$*

It was shown in part 1 that an asymptotic form of the stability equations could be derived for $|R_r| \to \infty$.† For oscillatory modes, the equations can be put in the form

$$\left.\begin{aligned}&\{(D'^2-\beta'^2+D'-ia')(D'^2-\beta'^2)+\beta'^2Q'\}\,W_1'\\&\qquad+\beta'^2T'\{1-\max(\lambda',0)+\lambda'\,e^{-\eta}\}(D'^2-\beta'^2)\,W_4'=0,\\&\{(D'^2-\beta'^2+D'-ia')(D'^2-\beta'^2)+\beta'^2Q'\}\,W_4'-e^{-\eta}W_1'=0,\end{aligned}\right\}\tag{3.1}$$

where

$$\left.\begin{aligned}&\eta=|r-r_s|\,|R_r|/r_s,\quad D'=d/d\eta,\\&W'=\frac{2M'r_s^{R_r+2}}{\nu}\frac{u'}{V_0},\quad W_4'=\frac{1}{kr_s}\frac{1}{V_0r_s\mu_0\sigma}\frac{B_\theta'}{B_0}R_r^4,\\&a'=\frac{p}{\nu}\left|\frac{r_s}{R_r}\right|^2,\quad \beta'=k\left|\frac{r_s}{R_r}\right|,\quad Q'=\frac{\sigma B_0^2}{\nu\rho}\left|\frac{r_s}{R_r}\right|^2.\end{aligned}\right\}\tag{3.2}$$

For outward radial flow

$$\left.\begin{aligned}&T'=T_2/\{R_r^4F(\lambda')\},\quad \lambda'=(V_s/V_\infty)-1,\\&F(\lambda')=\begin{cases}1+\lambda' & (\lambda'\geqslant-0{\cdot}5)\\-1/(4\lambda') & (\lambda'\leqslant-0{\cdot}5)\end{cases}\end{aligned}\right\}.\tag{3.3}$$

For inward radial flow,

$$T'=T_1/R_r^4,\quad \lambda'=1-(V_\infty/V_s);\tag{3.4}$$

r_s, V_s are the radius and tangential velocity of the suction cylinder and V_∞ is the tangential velocity at the outer edge of the suction boundary layer. For rotating cylinder flow, V_∞ is the tangential velocity of the injection cylinder extrapolated to the suction cylinder as a free vortex; i.e. $V_\infty = V_i r_i/r_s$, where r_i, V_i are the radius and tangential velocity of the injection cylinder. We have defined λ' so that for outward radial flow $\lambda' \leqslant 0$ for all cases of interest (i.e. for all cases where Rayleigh's criterion predicts instability), while for inward radial flow $\lambda' \geqslant 0$.

The boundary conditions at the suction wall are

$$W_1'=D'W_1'=D'W_4'=(D'^2-\beta'^2)\,W_4'=0\quad\text{at}\quad\eta=0;\tag{3.5}$$

and far from the wall,

$$W_1'\quad\text{and}\quad W_4'\to0\quad\text{as}\quad\eta\to\infty.\tag{3.6}$$

3.2. *Results for $|R_r| \to \infty$*

Figures 11–13 present the critical Taylor number, wave-number and frequency for the $|R_r| \to \infty$ theory as a function of Q' for $\lambda' = -1$ and $\lambda' = +1$, corresponding to a non-rotating outer cylinder with outward and inward radial flow, respectively. For very small Q', the critical modes are stationary and the Taylor

† As emphasized in part 1, the limit $|R_r| \to \infty$ cannot be taken too literally since we have assumed at the outset that $P_m|R_r|$ is small. Since $P_m \leqslant 10^{-6}$ for most fluids and the asymptotic theory will be shown to be a fair approximation for $|R_r| \geqslant 15$, there should be a wide range of $|R_r|$ for which this theory gives useful results.

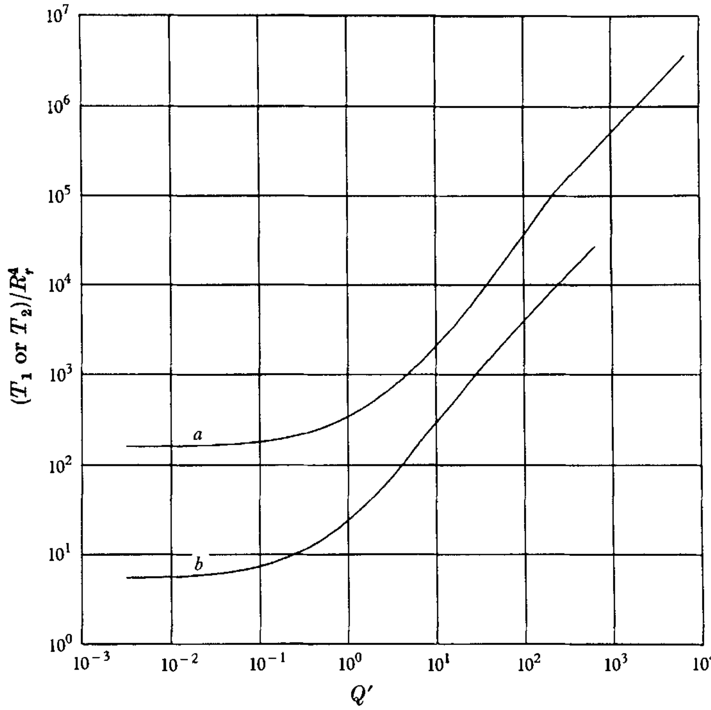

FIGURE 11. Critical Taylor number versus Hartmann number for $|R_r| \to \infty$. (*a*) $\lambda' = +1 \cdot 0$, read T_1 on ordinate; (*b*) $\lambda' = -1 \cdot 0$, read T_2 on ordinate.

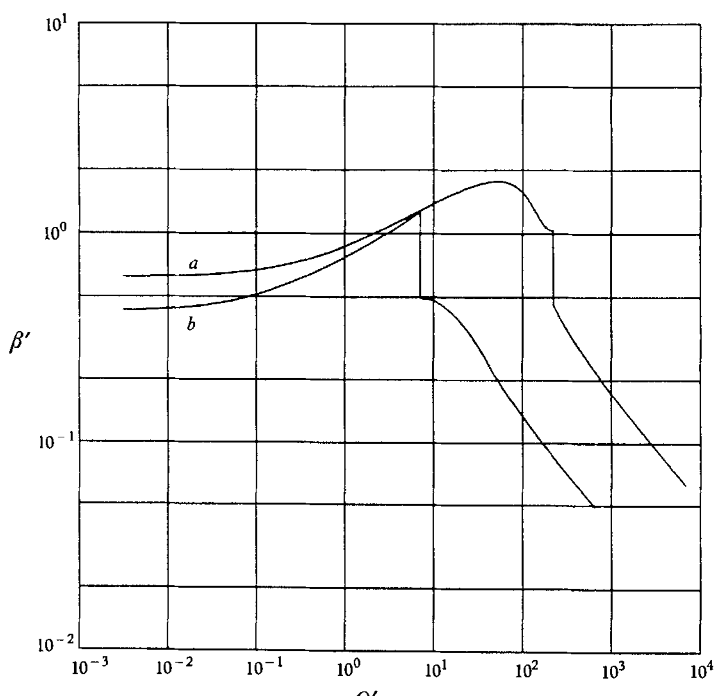

FIGURE 12. Critical wave-number versus Hartmann number for $|R_r| \to \infty$. (*a*) $\lambda' = +1 \cdot 0$; (*b*) $\lambda' = -1 \cdot 0$.

number and wave-number approach constant values. For very large Q', the frequency approaches a constant, the Taylor number is proportional to Q', and the wave-number is proportional to $Q'^{-\frac{1}{2}}$. This behaviour is entirely consistent with the earlier work on hydromagnetic stability between non-permeable cylinders by Chandrasekhar (1961) and, for the oscillatory mode case, by the present authors (1967*b*). One notable difference between the results for $\lambda' = -1$ and $\lambda' = +1$ is that the value of Q' required to produce oscillatory critical modes differs in the two cases by well over an order of magnitude. The difference is again believed to result from the difference in the shape of the tangential velocity profiles.

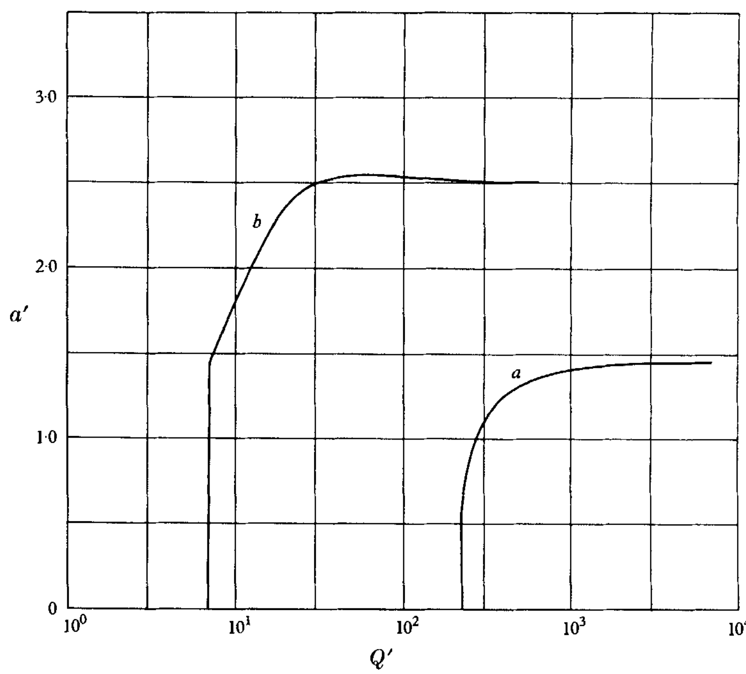

FIGURE 13. Critical frequency versus Hartmann number for $|R_r| \to \infty$. (*a*) $\lambda' = +1{\cdot}0$; (*b*) $\lambda' = -1{\cdot}0$.

The $|R_r| \to \infty$ results have also been plotted in figures 2 to 9 for comparison with the finite $|R_r|$ results. The Taylor number and wave-number obtained from the asymptotic calculations are in reasonably good agreement with the finite $|R_r|$ results for values of $|R_r|$ greater than about 15, although the final approach of the two sets of results is rather slow. The agreement in the critical frequency is somewhat poorer. In figure 10, the asymptotic frequency curve is not shown since the critical modes are oscillatory only at very low values of the radial Reynolds number where the asymptotic theory cannot be expected to apply.

3.3. $|R_r| \to \infty$ *and* $Q' \to \infty$

Chandrasekhar (1961) has shown that it is also possible to make use of the asymptotic behaviour with respect to Q' illustrated in figures 11–13 to simplify the stability equations further. Following Chandrasekhar (1961), equations (3.1) become

$$\left.\begin{aligned}&\{(D'^2+D'-ia')D'^2+Q'_\infty\}W'_1+T'_\infty\{1-\max(\lambda',0)+\lambda' e^{-\eta}\}D'^2W'_4=0,\\&\{(D'^2+D'-ia')D'^2+Q'_\infty\}W'_4-e^{-\eta}W'_1=0,\end{aligned}\right\} \quad (3.7)$$

where $Q'_\infty = \beta'^2 Q'$, $T'_\infty = \beta'^2 T'$ and we have taken the limit as $Q'\to\infty$ or $\beta'\to 0$. The boundary conditions become

$$W'_1 = D'W'_1 = D'W'_4 = D'^2W'_4 = 0 \quad \text{at} \quad \eta = 0, \quad (3.8)$$

$$W'_1,\, W'_4 \to 0 \quad \text{as} \quad \eta\to\infty. \quad (3.9)$$

We have solved equations (3.7) for several values of λ'. The resulting asymptotic forms are given in table 1 and are consistent with figures 11–13. The value of the critical frequency for $\lambda' = 1{\cdot}0$ is seen to be much less than the other frequencies

λ'	Wave-number	Taylor number	Frequency
$-1{\cdot}0$	$\beta' \sim 1{\cdot}23\, Q'^{-\frac{1}{2}}$	$T_2/R_r^4 \sim 167{\cdot}27\, Q'$	$a' \sim 2{\cdot}49$
$-0{\cdot}5$	$\beta' \sim 1{\cdot}37\, Q'^{-\frac{1}{2}}$	$T_2/R_r^4 \sim 149{\cdot}07\, Q'$	$a' \sim 2{\cdot}51$
$0{\cdot}0$	$\beta' \sim 1{\cdot}53\, Q'^{-\frac{1}{2}}$	$(T_1 \text{ or } T_2)/R_r^4 \sim 133{\cdot}98\, Q'$	$a' \sim 2{\cdot}53$
$0{\cdot}5$	$\beta' \sim 1{\cdot}83\, Q'^{-\frac{1}{2}}$	$T_1/R_r^4 \sim 220{\cdot}91\, Q'$	$a' \sim 2{\cdot}52$
$1{\cdot}0$	$\beta' \sim 5{\cdot}14\, Q'^{-\frac{1}{2}}$	$T_1/R_r^4 \sim 522{\cdot}15\, Q'$	$a' \sim 1{\cdot}47$

TABLE 1. Asymptotic results for both $|R_r|$ and Q' large

given in the table. In fact, the critical frequency is decreasing very rapidly as λ' approaches $1{\cdot}0$ and actually goes to zero at a value of λ' slightly greater than $1{\cdot}0$. (Values of $|\lambda'| > 1$ correspond to counter-rotating cylinders.) It is known definitely that at $\lambda' = 1{\cdot}06$ the critical mode remains stationary even in the limit as $Q'\to\infty$. Thus, the very high value of Q' required to produce oscillatory modes for $\lambda' = 1{\cdot}0$, which is shown in figure 13 and was noted earlier, seems to be related to the fact that $\lambda' = 1{\cdot}0$ is very close to a case for which oscillatory modes do not occur at all.

4. Conclusions

When $|R_r|$ is finite and the outer cylinder is stationary, oscillatory modes of instability occur for sufficiently large values of the Hartmann number. They lower the critical Taylor number and must be considered to obtain the correct stability criterion. With a constant Hartmann number, the distortions in the shape of the primary tangential velocity profile produced by small amounts of outward radial flow seem to encourage the formation of oscillatory modes, while inward radial flow seems to inhibit the oscillatory modes. For both inward and outward radial flow, however, as $|R_r|$ becomes very large the disturbance becomes confined to a thin boundary layer on the suction cylinder, and the Hartmann number based on the boundary-layer thickness is eventually reduced to a point where a transition back to stationary critical modes occurs.

In the limiting case of $|R_r| \to \infty$ with a stationary outer cylinder, the critical modes are stationary when Q', the Hartmann number based on boundary-layer thickness, is small and oscillatory when Q' is large. The behaviour of the critical parameters at very large values of Q' is qualitatively consistent with the earlier results for flow between nonpermeable perfectly conducting cylinders.

In the limiting case where both $|R_r|$ and Q' are very large, oscillatory critical modes are found for all cases of co-rotating cylinders for both inward and outward radial flow. For inward radial flow and counter-rotating cylinders, however, there is a range of cylinder rotation rates for which oscillatory critical modes do not occur.

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